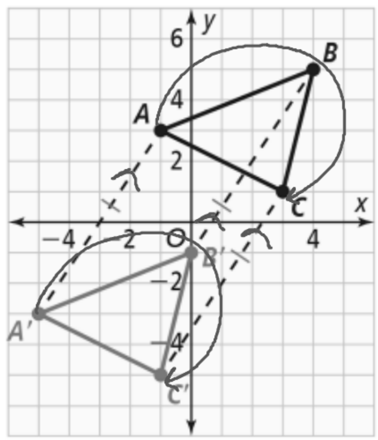
Translations

A translation is a transformation in a plane that maps all points of a preimage the same distance and in the same direction.

The translation of $\triangle ABC$ by x units along the x-axis and by y units along the y-axis can be written as $T_{\langle x,\, y\rangle}(\triangle ABC) = \triangle A'B'C'$.



A translation has the following properties:

If $T_{\langle x,\, y\rangle}(\triangle ABC) = \triangle A'B'C'$, then

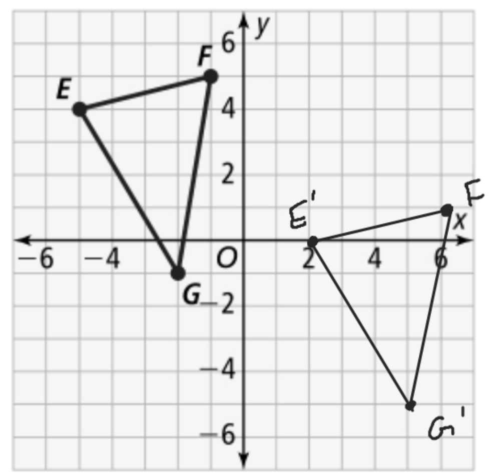
- $\overline{AA'} \parallel \overline{BB'} \parallel \overline{CC'}$.
- $\overline{AA'} \cong \overline{BB'} \cong \overline{CC'}$.
- $\triangle ABC$ and $\triangle A'B'C'$ have the same orientation.

A translation is a rigid motion, so length and angle measure are preserved.

What is the graph of $T_{\langle 7, -4\rangle}(\triangle EFG) = \triangle E'F'G'$?

Right

Down 4



$E(-5, 4) \rightarrow E'(-5+7,\ 4-4)$

$(2, 0)$

$F(-1, 5) \rightarrow F'(-1+7,\ 5-4)$

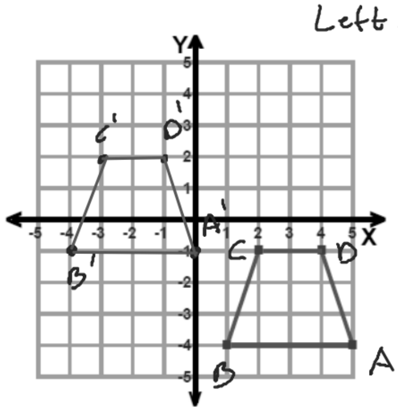
$(6, 1)$

$G(-2, -1) \rightarrow G'(-2+7,\ -1-4)$

$(5, -5)$

What is the graph of $T_{\langle -5,3 \rangle}$

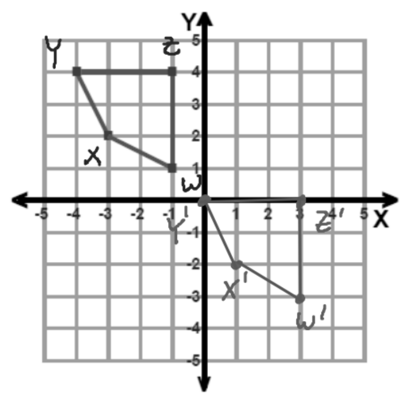
Left 5 up 3



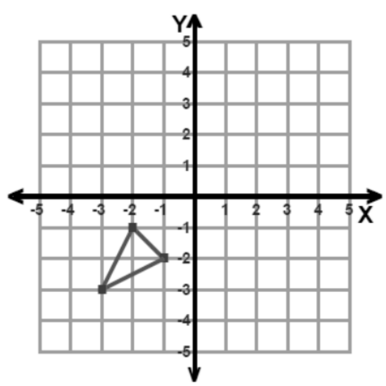
What is the graph of $T_{\langle 4,-4 \rangle}$

Right 4

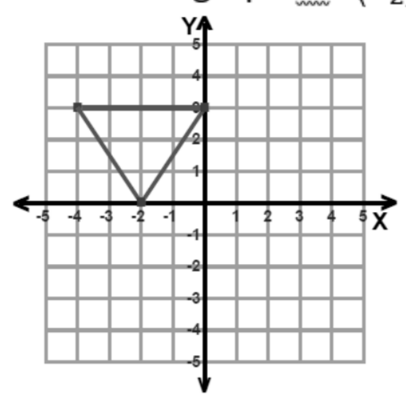
Down 4



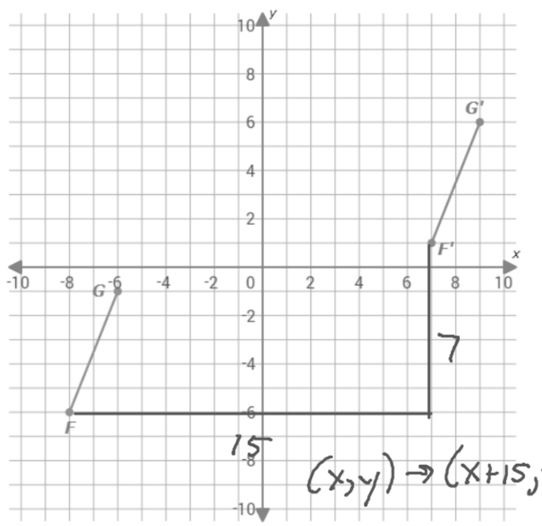
What is the graph of $T_{\langle 2,5 \rangle}$



What is the graph of $T_{\langle -2,-4 \rangle}$

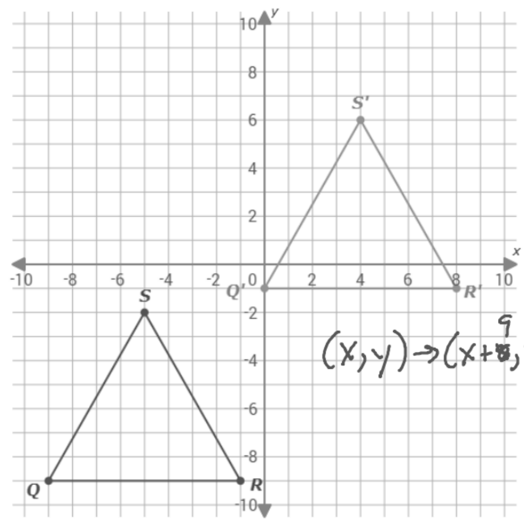


$\overline{F'G'}$ is a translation of $\overline{FG}$. Write the translation rule.



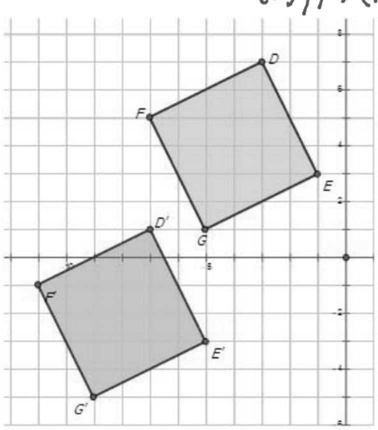
$(x,y) \to (x+15, y+7)$

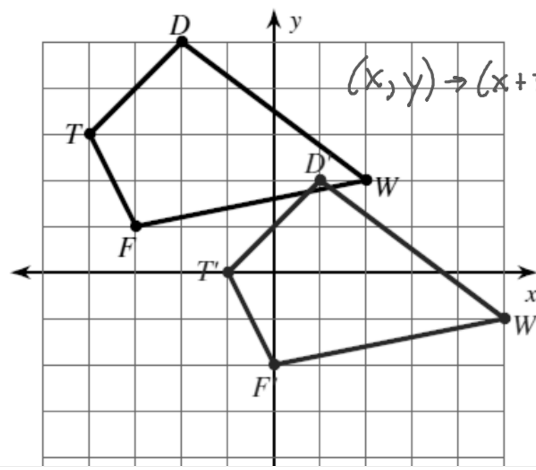
$\triangle Q'R'S'$ is a translation of $\triangle QRS$. Write the translation rule.



$(x,y) \to (x+\cancel{8}\,9, y+8)$

$(x,y) \to (x-4, y-6)$





$(x,y) \to (x+3, y-3)$

A **composition of rigid motions** is a transformation with two or more rigid motions in which the second rigid motion is performed on the image of the first rigid motion.

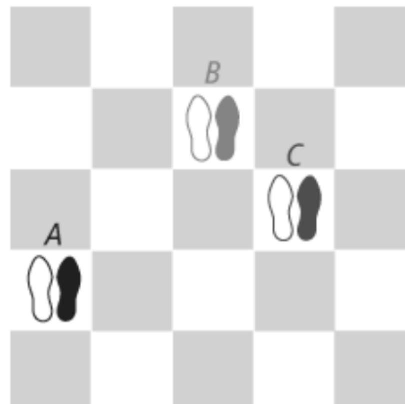
Step 1 Translate $\triangle ABC$ left 2 units and up 5 units.

$$A''B''C'' \leftarrow (R_\ell \circ \underset{\substack{\downarrow \\ A'B'C'}}{T_{\langle -2,\, 5\rangle}})(\triangle ABC)$$

This notation uses a small open circle to indicate a composition of rigid motions on $\triangle ABC$.

Step 2 Reflect $\triangle A'B'C'$ across line ℓ.

In learning a new dance, Kyle moves from position A to position B and then to position C. What single transformation describes Kyle's move from position A to position C?



SOLUTION

3. What is the composition of the transformations written as one transformation?

a. $T_{\langle 3, -2\rangle} \circ T_{\langle 1, -1\rangle}$ $\rightarrow$ $T_{\langle 4, -3\rangle}$

3. What is the composition of the transformations written as one transformation?

b. $T_{\langle -4, 0\rangle} \circ T_{\langle -2, 5\rangle}$ $\rightarrow$ $T_{\langle -4+(-2),\ 0+5\rangle}$

$T_{\langle -6, 5\rangle}$

4. Suppose n is the line with equation $y = 1$. Given $\triangle DEF$ with vertices $D(0,\ 0)$, $E(0,\ 3)$, and $F(3,\ 0)$, what translation image is equivalent to $(R_n \circ R_{x\text{-axis}})(\triangle DEF)$?

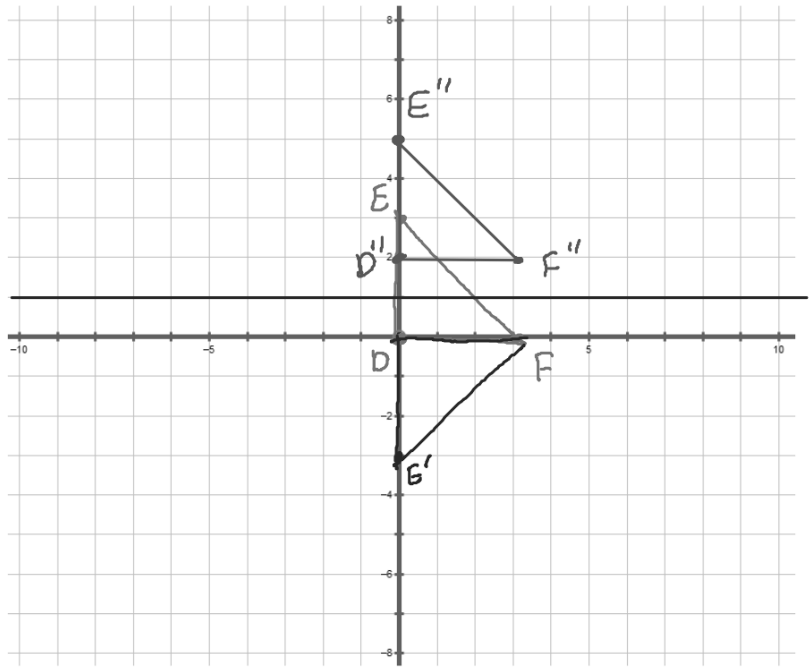
$R_{x\text{-axis}} - (x,y) \to (x,-y)$

$(x,y) \to (x,\ 2(1)-y)$

$D'\ (0,0)$ $\quad$ $D''\ (0,2)$

$E'\ (0,-3)$ $\quad$ $E''\ (0,5)$

$F'\ (3,0)$ $\quad$ $F''\ (3,2)$



A translation is a composition of reflections across two parallel lines.

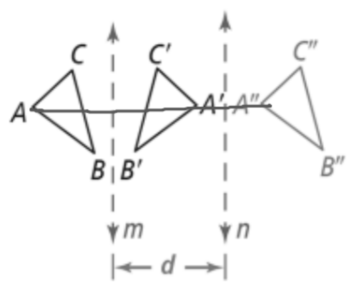
- Both reflection lines are perpendicular to the line containing a preimage point and its corresponding image point.
- The distance between the preimage and the image is twice the distance between the two reflection lines.

PROOF: SEE EXAMPLE 5.

If... $T(ABC) = A''B''C''$

$AA'' = BB'' = CC'' = 2d$

$AA'' = 2d$



Then... $(R_n \circ R_m)(ABC) = A''B''C''$